Monopoly and elasticity

A firm monopolizes a market with the following demand: $Q_d = 25 - 0.25P$, the cost conditions of this firm are: $CV_{me} = Q$ and $CF = \$100$. Determine and graph:

1. How many units will it produce and at what price?
2. The entrepreneurial profit.
3. The price elasticity of demand at the equilibrium point.

Solution

1. First, we clear the inverse demand: $P = 100 - 4Q$. On the other hand, given the average variable cost, we can obtain the variable cost: $CV = Q^2$. The profits are:

$$B = P * Q - TC = (100 - 4Q)Q - (Q^2 + 100)$$

We calculate the first-order condition:

$$\frac{\partial B}{\partial Q} = 100 - 8Q - 2Q = 0$$

$$100 = 10Q$$

$$Q = 10$$

And obtaining the price: $100 - 4 * 10 = 60 = P$

2. With the previous data, we obtain the profit:

$$B = 60 * 10 - 100 - 100 = 600 - 200 = 400$$

3. We obtain the price elasticity of the demand:

$$\frac{EQ_d}{EP} = \frac{\partial Q_d}{\partial P}\frac{P}{Q_d} = -0.25\frac{60}{10} = -1.5$$

In absolute value, the demand is greater than 1, therefore we say that it is elastic. The monopolist will never operate in the inelastic zone of the demand curve because in that zone, decreasing the quantity sold would increase total revenues and reduce costs, thereby increasing profits.